

On the bispectrum of COBE and WMAP

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ABSTRACT

The COBE-DMR 4-year maps displayed a strong non-Gaussian signal in the “inter-scale” components of the bispectrum: their observed values did not display the scatter expected from Gaussian maps. We re-examine this and other suggested non-Gaussian features in the light of WMAP. We find that they all disappear. Given that it was proved that COBE-DMR high noise levels and documented systematics could at most *dilute* the observed non-Gaussian features, we conclude that this dataset must have contained non-negligible undocumented systematic errors. It turns out that the culprit is a combination of QuadCube pixelization and data collected during the “eclipse season”.

Key words: cosmic microwave background - Gaussianity tests.

1 INTRODUCTION

The possibility of non-Gaussianity in the COBE-DMR 4 year maps led to a rather protracted story. Using “single- ℓ ” bispectrum analysis, Ferreira, Magueijo & Górski (1998) found strong evidence for non-Gaussianity in the anisotropy of the Cosmic Microwave Background (CMB) temperature. This detection was followed by similar claims by Novikov, Feldman and Shandarin (1998) and Pando, Valls-Gabaud & Fang (1998), and caused considerable consternation among theorists (see Kamionkowski and Jaffe (1998) for a discussion). Further work, however, showed that claims other than those based on the bispectrum could not be reproduced (Mukherjee, Hobson, Lasenby 2000). The bispectrum claims were confirmed by Bromley & Tegmark (1999).

Later Banday, Zaroubi & Górski (1999) cast serious doubts upon the cosmological origin of the observed signal. A systematic was identified which removed the observed “single- ℓ ” bispectrum signal – the so-called eclipse season data, which should never have been used. However, when an extension to “inter- ℓ ” bispectrum components was sought, a new non-Gaussian signal was found for inter- ℓ separations of $\Delta\ell = 1$ (Magueijo 2000). Specifically, their observed values were found to concentrate uncannily close to zero instead of displaying the scatter expected from Gaussian maps.

This signal could not be blamed on any systematic effects studied by Banday, Zaroubi & Górski (1999) or otherwise (Magueijo 2000). It was also proved that the instrument high noise levels could at most dilute the observed signal, in spite of the noise correlations and anisotropy

(Magueijo 2000). It was also found that the observed signal did not extend to higher inter- ℓ separations $\Delta\ell > 1$ (Sandvik & Magueijo 2001).

What shall we make of these claims in the light of the recent observations (Bennett & al 2003) by the Wilkinson Microwave Anisotropy Probe (WMAP)? In this paper we show that they don’t survive the new data, which displays consistency with Gaussianity on the angular scales probed by COBE. This might seem at first suprising, given that the WMAP data is considerably less noisy than the COBE-DMR dataset, and that noise can at most hide a non-Gaussian signal. However one should never forget the issue of systematics. Even though documented COBE-DMR systematics were shown not to correlate with the observed non-Gaussian signal, the new data allows us to cast a new look at the problem. We find that a highly non-subtle combination of QuadCube pixelization systematics and the “eclipse data” are to be blamed for the observed effect.

This, we believe, closes the story. The moral is clear: care must be exercised regarding similar claims currently being made with WMAP.

2 THE BISPECTRUM

We start by reviewing some results and definitions pertaining to the bispectrum. Given a full-sky map, $\frac{\Delta T}{T}(\mathbf{n})$, this may be expanded into Spherical Harmonic functions:

$$\frac{\Delta T}{T}(\mathbf{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\mathbf{n}) \quad (1)$$

The coefficients $a_{\ell m}$ may then be combined into rotationally invariant multilinear forms (see Magueijo, Ferreira, and Górski (1998) for a possible

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algorithm). The most general cubic invariant is the bispectrum, and is given by

$$\hat{B}_{\ell_1 \ell_2 \ell_3} = \frac{\begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}^{-1}}{(2\ell_1 + 1)^{\frac{1}{2}}(2\ell_2 + 1)^{\frac{1}{2}}(2\ell_3 + 1)^{\frac{1}{2}}} \times \sum_{m_1 m_2 m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \quad (2)$$

where the (...) is the Wigner $3J$ symbol. The proportionality constant is usually chosen in order to enforce a roughly constant cosmic variance. In Ferreira, Magueijo & Górski (1998) the choice was made $\ell_1 = \ell_2 = \ell_3$, leading to the “single- ℓ ” bispectrum $\hat{B}_\ell = B_{\ell\ell\ell}$. Other bispectrum components are sensitive to correlations between different scales. Selection rules require that $\ell_1 + \ell_2 + \ell_3$ be even. The simplest chain of correlators is therefore $\hat{A}_\ell = B_{\ell-1 \ell \ell+1}$ – the “inter- ℓ bispectrum” – and this was studied in Magueijo (2000). Other components, involving more distant multipoles, may be considered (Sandvik & Magueijo 2001) but they are very likely to be dominated by noise; it is natural to assume that possible non-Gaussian inter-scale correlations decay with ℓ separation.

We shall therefore consider ratios

$$I_\ell^3 = \frac{\hat{B}_\ell}{(\hat{C}_\ell)^{3/2}} \quad (3)$$

and

$$J_\ell^3 = \frac{\hat{A}_\ell}{(\hat{C}_{\ell-1})^{1/2}(\hat{C}_\ell)^{1/2}(\hat{C}_{\ell+1})^{1/2}} \quad (4)$$

where $\hat{C}_\ell = \frac{1}{2\ell+1} \sum_m |a_{\ell m}|^2$. These quantities are dimensionless, and therefore less dependent upon the power spectrum. They are also invariant under rotations and parity.

The theoretical importance of the bispectrum as a non-Gaussian qualifier has been recognized in a number of publications (Luo (1994), Peebles (1998), Spergel & Goldberg (1999), Goldberg & Spergel (1999), Wang & Kamionkowski (2000)). Kogut et al (1996) measured the pseudocollapsed and equilateral three point function of the DMR four year data. The bispectrum may be regarded as the Fourier space counterpart of the three point function.

3 BISPECTRUM ANALYSIS OF WMAP DATA

The WMAP mission (Bennett & al 2003) was designed to make full sky CMB maps with unprecedented accuracy. There are ten differencing assemblies (DAs) in total, four in the W band at 94 GHz, two V band at 61 GHz, two Q band at 41 GHz, one Ka band at 33 GHz, and one K band at 23 GHz. The K and Ka bands are dominated by galactic emission and therefore neglected for cosmological analysis. The maps are made using the HEALPix¹ format with $n_{\text{side}}=512$ (Gorski, Hivon & Wandelt 1998; Gorski & al 1999). The total number of pixels in each map is $12 \times n_{\text{side}}^2 = 3,145,728$. We use the coadded sum map of the Q, V and W maps,

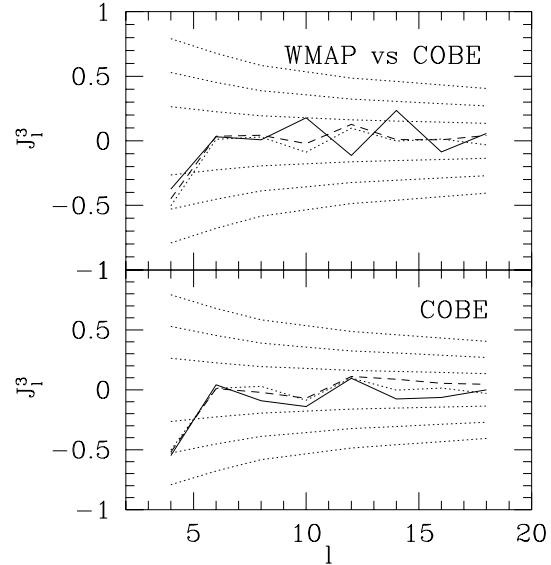


Figure 1. The COBE inter- ℓ non-Gaussian signal (bottom), using three different data renditions: HEALPix (solid line) and Quad-Cube ecliptic (dashed) and galactic (dotted). In the top panel we have re-reproduced the COBE galactic Quad-cube data (dotted), and superposed a COBE map in the same pixelization but without the eclipse data (dashed), and the WMAP results (solid). The continuum dotted lines represent 1, 2 and 3 sigma contours for the bispectrum expected from Gaussian maps.

$$T = \frac{\sum_{i=3}^{10} T_i / \sigma_{0,i}^2}{\sum_{i=3}^{10} 1 / \sigma_{0,i}^2} \quad (5)$$

where T_i is the sky map for the DA i with the foreground galactic signal subtracted, and $\sigma_{0,i}^2$ is the noise per observation for DA i , whose values are given by Bennett & al (2003). We use the publicly available ‘foreground cleaned’ maps, where the Galactic foreground signal, consisting of synchrotron, free-free, and dust emission, was removed using the 3-band, 5-parameter template fitting method described in Bennett et al. (2003). We then use the Kp0 mask to cut the Galactic plane emission and the known point sources (Bennett et al. (2003)), retaining 76.8% of the sky.

The monopole and dipole are removed and we perform a harmonic analysis of the map obtaining the $a_{\ell m}$ up to $l = 20$. This is performed using the FORTRAN utility ANAFast, available in the HEALPix package. We then evaluate (4) and the J_l^3 obtained for the WMAP data are compared to the distributions $P(J_l^3)$ obtained from Gaussian simulations subject to the appropriate beam, galactic mask and noise. These simulations take as input the LCDM power-law primordial power spectrum fit to the WMAP, CBI and ACBAR data (Bennett et al. (2003)). The random fields were generated using the utility SYNFAST of the HEALPix package. The distributions so obtained do not vary significantly from those obtained previously for COBE.

4 RESULTS AND DISCUSSION

In Fig. 1 we plot the inter- ℓ bispectrum of several renditions of the COBE data, and of WMAP. The original

¹ The HEALPix website is <http://www.eso.org/science/healpix>

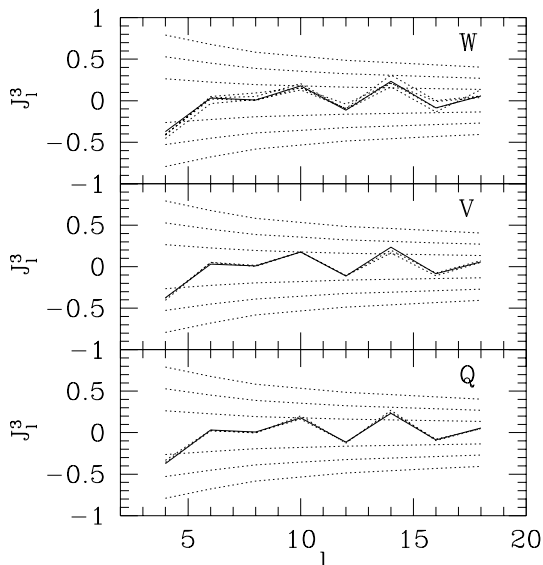


Figure 2. The inter- ℓ spectrum for the various WMAP Q, V, and W bands (dotted lines). The solid line on all panels is the spectrum for the co-added maps.

COBE-DMR non-Gaussian inter- ℓ signal (Magueijo 2000) was found using QuadCube pixelization. Possible deficiencies of this scheme were evaluated by aligning the pixelization system with galactic and ecliptic coordinates. The J_l^3 in both frames are plotted in the bottom panel of Fig. 1. The dotted contours represent 1, 2 and 3 sigma lines for the bispectrum arising from Gaussian realizations.

One would expect to see one in three coefficients lying outside the 1-sigma contour. Instead, for COBE-DMR maps we see a very close alignment with the peak of the distribution. This gives a reduced chi squared $X^2 = 0.14$ and $X^2 = 0.22$ for data in galactic and ecliptic pixelization, respectively. Computing the distributions $P(X^2)$ leads to $P(X^2 > 0.14) = 0.998$ (and $P(X^2 > 0.22) = 0.985$) for maps in galactic (ecliptic) pixelization. These are the confidence levels for rejecting Gaussianity on the grounds of the COBE bispectrum.

As can be seen from the upper panel in Fig. 1 the WMAP bispectrum does not have such an obvious lack of scatter. Indeed it leads to $X^2 = 0.59$, consistent with Gaussianity.

What can be the origin of this discrepancy? It is at once clear that we are not comparing like with like. The WMAP project used the HEALPix pixelization scheme, and the tests made for the impact of QuadCube upon the COBE map may not be conclusive. For example there may be an isotropic systematic effect, present no matter how one orients the coordinate system. To address this in Fig. 1 we plotted (solid line in bottom panel) the J_l^3 from COBE 4 year maps rendered in HEALPix. We found $X^2 = 0.26$, reducing the confidence level to 96%. Hence the COBE/WMAP discrepancy can partly be blamed on a poor pixelization scheme, but this is not enough: even in the HEALPix rendition COBE is much more non-Gaussian than WMAP.

Could we have missed the non-Gaussian signal in WMAP by looking at the wrong combination of frequen-

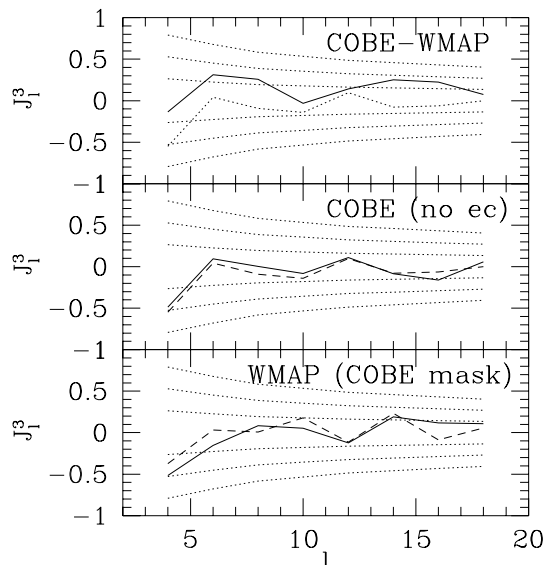


Figure 3. The bottom panel shows the J_l^3 for WMAP subject to the COBE mask (solid line) and to the Kp0 mask (dotted). The middle panel shows the J_l^3 for the COBE maps in HEALPix with (dashed) and without (solid) the eclipse data. The top panel (solid line) shows the inter- ℓ spectrum of a map obtained by subtracting WMAP to COBE (both with the COBE mask). We have added the COBE (HEALPix) bispectrum for reference.

cies? Equation (5) favours the Q band, which is the most contaminated by galactic emission. Galactic emissions have been proved to degrade non-Gaussian large angle signals (Magueijo 2000). However, as can be seen in Fig. 2 there is a remarkable consistency between the bispectrum observed in all WMAP channels, a tribute to the very high signal to noise, but also to the lack of galactic contamination. By way of contrast the COBE inter- ℓ non-Gaussian signal came mainly from the 53 GHz channel, which was also the least noisy channel.

A related possible source of discrepancy is the galactic mask. The Kp0 mask is significantly smaller than the extended galactic cut used in COBE (Banday & al 1997). Could the extra regions near the galactic plane hide a non-Gaussian signal? As the bottom panel in Fig. 3 shows this is not the case. Subjecting the WMAP data to the extended cut used by the COBE team in fact increases the WMAP chi squared to $X^2 = 0.64$.

It would therefore appear that nothing has been missed, and that the WMAP bispectrum on “COBE” large angular scales is indeed consistent with Gaussianity. Given that the higher noise levels in the COBE maps can at most dilute a non-Gaussian signal (a fact proved in Magueijo (2000) even after taking noise correlations into account), we may conclude that a systematic error is behind the COBE non-Gaussian signal.

Nevertheless, identifying the culprit is far from obvious: no documented COBE systematic mimics the observed inter- ℓ signal. The effects of the eclipse data (Banday, Zaroubi & Górski 1999) on the J_l^3 , for example, are assessed in the top panel of Fig. 1. They give $X^2 = 0.18$, leading a confidence level for rejecting Gaussianity of 99.2%.

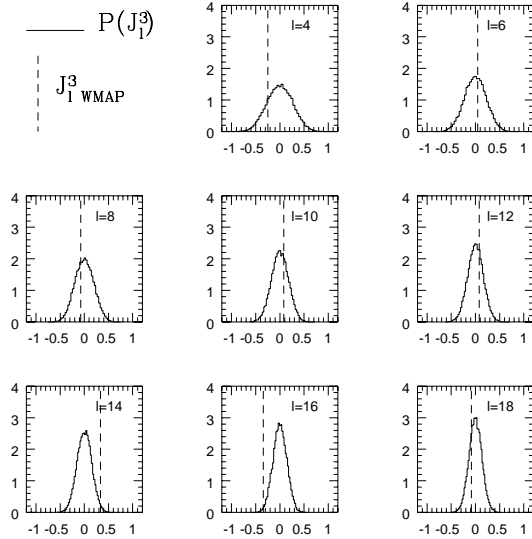


Figure 4. The vertical thick dashed line is the value of the observed J_l^3 . The solid line histogram is the pdf of the J_l^3 obtained from 4020 Gaussian simulations of the sky with noise and the Kp0 mask. The dashed line histogram superimposed is the same pdf obtained with the COBE's extended galactic cut and DMR noise.

It turns out, however, that if we reject data collected during the eclipse season *and* construct a DMR 4-year map in HEALPix the puzzle is solved. This is shown in the middle panel in Fig. 3, where we have plotted the J_l^3 inferred from such (co-added) maps. Although the difference might look subtle, there is quite a substantial difference around $\ell = 16$. In fact the reduced chi squared is now $X^2 = 0.42$, consistent with Gaussianity.

Curiously the single- ℓ non-Gaussian signal found by Ferreira, Magueijo & Górski (1998) results mainly from I_ℓ^3 at $\ell = 16$. This stops being a severe deviant once the eclipse data is excluded. It would now appear that the same happens for the J_ℓ^3 , but only after a better pixelization scheme is introduced.

5 CONCLUSIONS

The conclusion is that a combination of pixelization effects and the “eclipse” systematic is behind the inter- ℓ non-Gaussian effect previously reported for COBE maps. Curiously if we take a difference map (COBE minus WMAP) the anomalous alignment of the J_l^3 is not present (see top panel of Fig. 3). On the scales we are considering such maps picture COBE noise, plus COBE systematics, minus WMAP systematics (assumed to be small). Thus one would expect the COBE J_l^3 signal to be enhanced in the difference map. The fact that it is not results from the non-linearity of the statistic being used, plus the subtle interplay of signal and systematic via the QuadCube pixelization scheme.

In this paper we have concentrated on J_ℓ^3 , because the single scale bispectrum anomalies have been explained long ago. However we have checked that no new anomalies on large angular scales emerge in the WMAP data.

We reserve to a future publication a complete study of the bispectrum of WMAP of smaller angular scales. To our mind the work of Komatsu & al (2003) is just the beginning.

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